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SELF-SELECTIVE SOCIAL CHOICE FUNCTIONS VERIFY ARROW AND GIBBARD-SATTERTHWAITE THEOREMS

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This paper introduces a new notion of consistency for social choice functions, called self-selectivity, which requires that a social choice function employed by a society to make a choice from a given alternative set it faces should choose itself from among other rival such functions when it is employed by the society to make this latter choice as well. A unanimous neutral social choice function turns out to be universally self-selective if and only if it is Paretian and satisfies independence of irrelevant alternatives. The neutral unanimous social choice functions whose domains consist of linear order profiles on nonempty sets of any finite cardinality induce a class of social welfare functions that inherit Paretianism and independence of irrelevant alternatives in case the social choice function with which one starts is universally self-selective. Thus, a unanimous and neutral social choice function is universally self-selective if and only if it is dictatorial. Moreover, universal self-selectivity for such functions is equivalent to the conjunction of strategy-proofness and independence of irrelevant alternatives or the conjunction of monotonicity and independence of irrelevant alternatives again.

KEYWORDS: Self-selectivity, dictatorship, strategy-proofness, monotonicity, independence of irrelevant alternatives.

1. INTRODUCTION

WE IMAGINE A SITUATION where a society faced with a finite nonempty set A of alternatives is also to choose the social choice function (SCF) according to which the choice from A will be made. Our society here is assumed to be endowed with a preference profile on A and to have a finite nonempty set $\mathscr{A}$ of SCFs available to make its choice of an SCF. If each agent's preferences on A are represented by a linear order, then our agents will rank the available SCFs in $\mathscr{A}$ in accordance with what alternatives from A are chosen under these at the existing preference profile on A, yielding a complete preorder[2] on $\mathscr{A}$ for each agent relative to which two SCFs leading to the same alternative are regarded as equivalent, of course. In order to answer the question of which SCF the given society will choose from $\mathscr{A}$, we still need to specify the social choice rule that will be employed by our society in making this choice. A natural question that arises now is which ones from among the available SCFs will choose themselves when they are employed as the rules according to which the choice from $\mathscr{A}$ will be carried out. The answer to this question clearly depends upon the composition of $\mathscr{A}$ as well as the society's existing preference profile on the set A of underlying alternatives.

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[2]A complete preorder ρ on a nonempty set B is a binary relation on B that is complete (i.e., $x \rho y$ or $y \rho x$ for all $x, y \in B$) and transitive on B.

If an SCF being used by our society for making its choice from A does not get chosen by itself in the presence of other available SCFs when it comes to choosing the SCF itself, then ascribing this phenomenon to a certain lack of consistency on the part of the SCF in question should not be met with surprise. For, in such a case, it will be according to the very rationale of its own that this SCF gets beaten by some other available SCF. We are now ready to introduce the notion of self-selectivity[3] for an SCF, delaying its precise definition to the next section. Given a preference profile R on A and a set $\mathscr{A}$ of available SCFs, we will regard an SCF in $\mathscr{A}$ as self-selective relative to $\mathscr{A}$ at R if it chooses itself from $\mathscr{A}$ at some preference profile on $\mathscr{A}$ induced by the profile R on A as roughly described above and again to be made precise later. If an SCF is self-selective at R relative to any finite set $\mathscr{A}$ of SCFs to which it belongs, then we will say that it is self-selective at R. Finally, an SCF will be said to be universally self-selective if it is self-selective at each profile R on the underlying set of alternatives. The main question this paper addresses is the characterization of universally self-selective SCFs.

Notice that, even if we confine ourselves to a fixed set of alternatives, the set $\mathscr{A}$ of available SCFs is allowed to have any cardinality so long as it is finite and nonempty. Thus, for the notion of self-selectivity to make sense, preference profiles induced on $\mathscr{A}$ should belong to the domain of our SCF no matter what the (finite and positive) size of this set is. Restricting ourselves to neutral SCFs only allows us to consider each initial segment of natural numbers as a representative of alternative sets of the corresponding size. Keeping our society fixed throughout the discussion, we will further assume that the agent's preferences are always linear orders. So, the domain of an SCF in the present context will consist of all linear order profiles on the initial segments of natural numbers, where each such profile is, of course, mapped onto a member of the initial segment on which this profile is defined.

As we have noted before, however, a preference relation induced on a set of available SCFs by a linear order on the set of underlying alternatives will not, in general, be anti-symmetric,[4] for two different SCFs may lead to the same alternative at some linear order profiles. Thus, the preference profile induced on the set $\mathscr{A}$ of available SCFs by a linear order profile on the original set A of alternatives will only be a complete preorder profile. We regard a linear order as compatible with a given complete preorder if it can be obtained from this preorder by breaking ties in the indifference classes of the preorder in some way. So, a linear order profile on A will lead to a class of linear order profiles on $\mathscr{A}$, each of whose components is compatible with the corresponding component of the complete preorder profile induced on $\mathscr{A}$. For an SCF to be self-selective,

[3]In earlier versions of this paper, the notion that we call self-selectivity here was referred to simply as consistency. Since the latter term is already being used in several different contexts, following the suggestion of an anonymous referee, we adopted the name of self-selectivity for the particular kind of consistency considered here.

[4]A binary relation ρ on a nonempty set B is *anti-symmetric* if and only if, for all $x, y \in B$, one has $x = y$ whenever $x \rho y$ and $y \rho x$.

we require the existence of one linear order profile on $\mathscr{A}$ belonging to this class at which our SCF chooses itself.[5]

Having summarized the kind of consistency we deal with in this paper, we now turn to its characterization. We first note that, given an SCF F, each linear order profile R on A leads to a choice function c_R that assigns to each nonempty subset B of A the singleton consisting of the image of the restriction of R to B under F. In general, there need not exist any complete preorder on A whose optimization over nonempty subsets of A will result in c_R, because c_R will not necessarily satisfy Houthakker's Axiom. If a neutral SCF is universally self-selective, however, then c_R turns out to satisfy Houthakker's Axiom for each linear order profile R on A. Associating with each such R the unique linear order on A leading to c_R yields a social welfare function (SWF). The SWF we thus obtain starting with the SCF F is clearly neutral, and it specifies an aggregation procedure for linear order profiles on finite nonempty alternative sets of any size. To see what properties of the initial SCF are inherited by the SWF to which it leads, we first show that a unanimous neutral SCF is universally self-selective if and only if it is Paretian and satisfies Independence of Irrelevant Alternatives (IIA). It is actually IIA of our SCF that makes the choice function c_R satisfy Houthakker's Axiom for each linear order profile R, whereby IIA also gets inherited by the SWF with which we end up. Moreover, an SWF to which a unanimous neutral and universally self-selective SCF leads also turns out to be Paretian. Thus, the restriction of such an SWF to linear order profiles on a finite alternative set with at least three members is shown to be dictatorial by Arrow's famous Impossibility Theorem (Arrow (1963)). This, in turn, implies the dictatoriality of the original SCF on its whole domain, including linear order profiles on doubleton sets as well. In summary, a unanimous neutral SCF turns out to be universally self-selective if and only if it is dictatorial. Once this result is reached, however, we trivially obtain further equivalences for a unanimous neutral SCF to be universally self-selective by the famous Gibbard-Satterthwaite (Gibbard (1973); Satterthwaite (1973)) and Müller-Satterthwaite (Müller and Satterthwaite (1977)) Theorems. That is, a unanimous neutral SCF is universally self-selective if and only if it is strategy-proof and satisfies IIA or, equivalently, it is (Maskin)-monotonic and satisfies IIA. Here again strategy-proofness as well as monotonicity are meant to hold on the entire domain of our SCF. Finally, to avoid misconceptions, we wish to emphasize once more that here we do not deal with a fixed alternative space, but each of our neutral SCFs prescribes what alternative gets chosen once a profile on a finite nonempty set is given, no matter what the size of that set is.

The only paper of which we know that has dealt with a similar consistency notion is Binmore (1975),[6] where an example for a three-element alternative set

[5]The requirement for the SCF to choose itself at *all* such induced linear order profiles on $\mathscr{A}$ turns out to be too strong in the sense that it renders universal self-selectivity a vacuous concept.

[6]Koray (1998) also deals with consistency in the sense of self-selectivity and rediscovers the Condorcet rule as the maximal self-selective social choice correspondence among neutral and top-majoritarian social choice rules.

is constructed to show that inconsistencies will arise at certain preference profiles if the society agrees on a "constitution of order 3" to settle all conflicts arising over any set of three alternatives unless the constitution is a dictatorship, anti-dictatorship, or collective apathy.[7] A constitution of order 3 in Binmore (1975) is nothing but a neutral social welfare function whose domain consists of all complete preorder profiles on a three-element alternative set. The reasons why anti-dictatorship and collective apathy are included among SWFs leading to no inconsistencies are that the SWFs there need not be unanimous, and preference relations are not restricted to linear orders. The crucial idea in Binmore (1975) that coincides with the basic notion behind self-selectivity here is that a society that has agreed upon a constitution of order 3 should use that same constitution when the alternative set consists of three such constitutions. Apart from the fact that we deal with unanimous SCFs defined on linear order profiles here, we essentially obtain Binmore's (1975) result as a corollary. For, in our setting, starting with a (unanimous) neutral universally self-selective SCF restricted to profiles on a k-element set turns out to be equivalent to taking a (Paretian) SWF satisfying IIA on such a domain as our point of departure.

In the next section, we introduce and define some basic notions. Section 3 reports our main results, followed by closing remarks in the last section.

2. BASIC NOTIONS

Let N be a finite nonempty society that will be kept fixed throughout the whole discussion. Let $\mathbb{N}$ stand for the set of natural numbers as usual, set $I_m = \{1, \ldots, m\}$ and denote the set of all linear orders on I_m by $\mathscr{L}(I_m)$ for each $m \in \mathbb{N}$. We will call a function

$$F\colon \bigcup_{m\in\mathbb{N}} \mathscr{L}(I_m)^N \to \mathbb{N}$$

a *social choice function* (SCF) if and only if, for each $m \in \mathbb{N}$ and each $R \in \mathscr{L}(I_m)^N$, one has $F(R) \in I_m$. The set of all social choice functions will be denoted by $\mathscr{F}$.

For each $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$, and every permutation σ_m on I_m, we define the permuted linear order profile R_{σ_m} on I_m through the following biconditional: For all $i \in N$, $k, l \in I_m$, $kR^i_{\sigma_m}l$ if and only if $\sigma_m(k)R^i\sigma_m(l)$. Now $F \in \mathscr{F}$ will be called *neutral* if and only if, for each $m \in \mathbb{N}$ and every permutation σ_m on I_m, one has

$$\sigma_m\big(F(R_{\sigma_m})\big) = F(R).$$

We will denote the set of all neutral SCFs by $\mathscr{N}$.

Neutrality of an SCF F will allow us to extend the domain of F to linear order profiles on any finite nonempty set in a natural fashion. Take any finite set A with $|A| = m \in \mathbb{N}$, where $|A|$ stands for the cardinality of A. Let $\mu\colon I_m \to A$ be

[7]An anti-dictatorship is a dictatorship with the dictator's preferences reversed. By collective apathy is meant a constant SWF where all alternatives are held indifferent.

a bijection (i.e. a one-to-one and onto function). Any linear order profile L on A induces a linear order profile L_μ on I_m like above, where, for all $i \in N$ and $k, l \in I_m$, one has $kL^i_\mu l$ if and only if $\mu(k)L^i\mu(l)$. Now it is clear that $\mu(F(L_\mu)) = \nu(F(L_\nu))$ for any two bijections $\mu, \nu: I_m \to A$ if F is neutral. We will simply define $F(L) = \mu(F(L_\mu))$, where μ is a bijection from I_m onto A.

To define the notion of self-selectivity for a neutral SCF, let us first see how a linear order profile $R \in \mathscr{L}(I_m)^N (m \in \mathbb{N})$ induces a preference profile on any given nonempty finite subset $\mathscr{A}$ of $\mathscr{N}$. Take any $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$ and any nonempty finite subset $\mathscr{A}$ of $\mathscr{N}$. Now we define the relations $R^i_{\mathscr{A}}(i \in N)$ on $\mathscr{A}$ through the following biconditional: For all $F, G \in \mathscr{A}$ and $i \in N$, $FR^i_{\mathscr{A}}G$ if and only if $F(R)R^iG(R)$. In other words, the agents in our society N endowed with the preference profile R on I_m evaluate the SCFs in $\mathscr{A}$ according to the outcomes in I_m these choose at R. Clearly, $R^i_{\mathscr{A}}(i \in N)$ is a complete preorder on $\mathscr{A}$ which is not anti-symmetric in case more than one SCF in $\mathscr{A}$ choose the same member in I_m. We call $R^N_{\mathscr{A}}$ *the preference profile on* $\mathscr{A}$ *induced by* R and simply denote it by $R_{\mathscr{A}}$.

Given a complete preorder ρ on a finite nonempty set A, a linear order λ on A will be said to be *compatible with* ρ if and only if, for all $x, y \in A$, $x\lambda y$ implies $x\rho y$. In other words, a linear order on A that can be obtained from ρ by breaking ties through linearly ordering the members in each indifference class of ρ in some way is considered to be compatible with ρ. Now, for each $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$ and every nonempty finite subset $\mathscr{A}$ of $\mathscr{N}$, we will set $\mathscr{L}(\mathscr{A}, R) = \{L \in \mathscr{L}(\mathscr{A})^N | L^i$ is a linear order on $\mathscr{A}$ compatible with $R^i_{\mathscr{A}}$ for each $i \in N\}$, where $\mathscr{L}(\mathscr{A})$ stands for the set of all linear orders on $\mathscr{A}$, and call $\mathscr{L}(\mathscr{A}, R)$ *the set of all linear order profiles on* $\mathscr{A}$ *induced by* R.

Finally, we are ready to define self-selectivity for a neutral SCF. Given $F \in \mathscr{N}$, $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$ and a finite subset $\mathscr{A}$ of $\mathscr{N}$ with $F \in \mathscr{A}$, we say that F is *self-selective at* R *relative to* $\mathscr{A}$ if and only if there exists some $L \in \mathscr{L}(\mathscr{A}, R)$ such that $F = F(L)$. Here we imagine that our society N faced with a set of alternatives I_m is also to choose the SCF that it will employ in making its choice from I_m from among the available SCFs in $\mathscr{A}$. For the SCF F chosen from $\mathscr{A}$ by N to be considered as consistent in the sense of self-selectivity, we require the existence of a linear order profile L on $\mathscr{A}$ compatible with our society's underlying preference profile R on the original set I_m of alternatives at which F does not choose some choice rule in $\mathscr{A}$ other than itself.

Moreover, we say that F is *self-selective at* R if and only if F is self-selective at R relative to any finite subset $\mathscr{A}$ of $\mathscr{N}$ with $F \in \mathscr{A}$. Finally, F is said to be *universally self-selective* if and only if F is self-selective at each $R \in \cup_{m \in \mathbb{N}} \mathscr{L}(I_m)^N$.

Before proceeding with the characterization of self-selective neutral unanimous SCFs, it seems useful to consider an example illustrating this new notion of consistency. Let us first give the definition of unanimity in our framework. An SCF $F \in \mathscr{N}$ is said to be *unanimous* if and only if, for all $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$, and $a \in I_m$,

$$\left[\forall_i \in N, \forall b \in I_m : aR^ib\right] \Rightarrow F(R) = a.$$

Now consider a society $N = \{\alpha, \beta, \gamma, \delta\}$ consisting of four agents. Let F_1 be the plurality function where all ties are broken in favor of α. Given any $m \in \mathbb{N}$ and $R \in \mathscr{L}(I_m)^N$, an outcome $a \in I_m$ is said to be a *Condorcet winner* at R if and only if, for all $b \in I_m \setminus \{a\}$, $|\{i \in N \mid aR^i b\}| \geq |N|/2 = 2$. In case the set of Condorcet winners at R is nonempty, we define $F_2(R)$ to be the Condorcet winner most preferred by α if m is odd, and the Condorcet winner most preferred by β if m is even; if there are no Condorcet winners at R at all, we set $F_2(R)$ equal to the top outcome of R^α. We let F_3 stand for the Borda function where ties are broken in favor of γ and the scoring vector employed on I_m is the standard one, namely $(m, m-1, \dots, 1)$, for each $m \in \mathbb{N}$. Finally, F_4 will denote the dictatorial SCF where δ is the dictator, i.e., F_4 assigns the top alternative of R^δ to each $R \in \cup_{m \in \mathbb{N}} \mathscr{L}(I_m)^N$. It is clear that F_1, F_2, F_3, and F_4 are all neutral and unanimous SCFs.

Now let us consider the linear order profile R on I_3 given through the following table:

R^α	R^β	R^γ	R^δ
2	1	3	1
1	3	2	2
3	2	1	3

First consider the case where the set $\mathscr{A}$ of available SCFs is $\{F_1, F_2, F_3\}$. We have $F_1(R) = 1$, $F_2(R) = 2$, and $F_3(R) = 1$. The complete preorder $R_{\mathscr{A}}$ on $\mathscr{A}$ induced by R is represented in the following table with boxed sets of alternatives indicating indifference classes:

$R^\alpha_{\mathscr{A}}$	$R^\beta_{\mathscr{A}}$	$R^\gamma_{\mathscr{A}}$	$R^\delta_{\mathscr{A}}$
F_2	$\boxed{F_1, F_3}$	F_2	$\boxed{F_1, F_3}$
$\boxed{F_1, F_3}$	F_2	$\boxed{F_1, F_3}$	F_2

Now $\mathscr{L}(\mathscr{A}, R)$ consists of 2^4 linear order profiles compatible with the above complete preorder profile in each component. The linear order profile L below is a member of $\mathscr{L}(\mathscr{A}, R)$:

L^α	L^β	L^γ	L^δ
F_2	F_3	F_2	F_3
F_3	F_1	F_3	F_1
F_1	F_2	F_1	F_2

Since $F_2(L) = F_2$ and $F_3(L) = F_3$, we conclude that both F_2 and F_3 are self-selective at R relative to $\mathscr{A}$. However, not only is it true that $F_1(L) = F_2 \neq F_1$, but we also have $F_1(L') \neq F_1$ for any $L' \in \mathscr{L}(\mathscr{A}, R)$ since, at each such L', F_2 is top-ranked by two members of N including α to whose favor all ties are broken under F_1.

Now consider the case where $\mathscr{A}' = \{F_2, F_3\}$. Here $\mathscr{L}(\mathscr{A}', R)$ consists of one member L_1 only, where

L_1^α	L_1^β	L_1^γ	L_1^δ
F_2	F_3	F_2	F_3
F_3	F_2	F_3	F_2

Now $F_2(L_1) = F_3 \neq F_2$ and $F_3(L_1) = F_2 \neq F_3$. Since $\mathscr{L}(\mathscr{A}', R) = \{L_1\}$, this means that neither F_2 nor F_3 is self-selective at R relative to $\mathscr{A}'$.

Finally, assume that our society's available set $\mathscr{A}''$ of SCFs is $\{F_3, F_4\}$. Noting that $F_4(R) = 1 = F_3(R)$, we see that $\mathscr{L}(\mathscr{A}'', R)$ contains all linear order profiles on $\mathscr{A}''$. Thus, the profile L_3 at which all agents in N top rank F_3 as well as the profile L_4 where everyone prefers F_4 to F_3 belongs to $\mathscr{L}(\mathscr{A}'', R)$. Clearly, $F_3(L_3) = F_3$ and $F_4(L_4) = F_4$. Thus, both F_3 and F_4 are self-selective at R relative to $\mathscr{A}''$. Actually, it is trivially true that F_4 is self-selective at any linear order profile in $\cup_{m \in \mathbb{N}} \mathscr{L}(I_m)^N$ relative to any finite set of unanimous neutral SCFs containing itself, i.e., F_4 is universally self-selective. Moreover, we have also seen that none of F_1, F_2, F_3 is universally self-selective. Our characterization result in the next section will tell us that the observations to which the example here leads are not accidental at all.

3. RESULTS

To state our results we need some further definitions. An SCF $F \in \mathscr{N}$ is called *Paretian* if and only if, for all $R \in \cup_{m \in \mathbb{N}} \mathscr{L}(I_m)^N$, $F(R)$ is Pareto optimal with respect to R. We say that an SCF $F \in \mathscr{N}$ satisfies *Independence of Irrelevant Alternatives* (*IIA*) if and only if, for all $m \in \mathbb{N}$ and $R \in \mathscr{L}(I_m)^N$, one has

$$[\varnothing \neq B \subset I_m, F(R) \notin B] \Rightarrow F(R) = F(R|_{I_m \setminus B}),$$

where $R|_{I_m \setminus B}$ denotes the restriction of R to $I_m \setminus B$.

THEOREM 1: *Let $F \in \mathscr{N}$ be a unanimous SCF. Now F is universally self-selective if and only if F is Paretian and satisfies IIA.*

PROOF: First note that, for any $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$, and $a \in I_m$, there exists some $H \in \mathscr{N}$ such that $H(R) = a$.

Now assume that F is universally self-selective. Take any $m \in \mathbb{N}$ and $R \in \mathscr{L}(I_m)^N$. Set $F(R) = a$. First suppose that there exists some $b \in I_m$ Pareto dominating a with respect to R. Take some $H \in \mathscr{N}$ such that $H(R) = b$. Now let $\mathscr{A} = \{F, H\}$. Clearly, $\mathscr{L}(\mathscr{A}, R) = \{L\}$, where HL^iF for all $i \in N$. On the one hand, $F(L) = F$ since F is self-selective at R relative to $\mathscr{A}$. On the other hand, $F(L) = H$ since F is unanimous, yielding a contradiction. Therefore F is Paretian.

To show that F also satisfies IIA, assume that B is a nomempty subset of I_m with $a \notin B$. For each $k \in I_m \setminus (B \cup \{a\})$, choose some $H_k \in \mathscr{N}$ with $H_k(R) = k$.

Set $\mathscr{A} = \{F\} \cup \{H_k \mid k \in I_m \setminus (B \cup \{a\})\}$. Since $G_1(R) \neq G_2(R)$ for any $G_1, G_2 \in \mathscr{A}$ with $G_1 \neq G_2$, we have $\mathscr{L}(\mathscr{A}, R) = \{L\}$ for some $L \in \mathscr{L}(\mathscr{A})^N$. On the one hand, $F(L) = F$ since F is self-selective at R relative to $\mathscr{A}$. On the other hand, defining a bijection $\sigma : I_m \setminus B \to \mathscr{A}$ by letting $\sigma(a) = F$ and $\sigma(k) = H_k$ for each $k \in I_m \setminus (B \cup \{a\})$, we see that $(R|_{I_m \setminus B})_{\sigma^{-1}} = L$. Thus, it follows by neutrality of F that

$$F = F(L) = F\big((R|_{I_m \setminus B})_{\sigma^{-1}}\big) = \sigma(F(R|_{I_m \setminus B})),$$

implying that

$$F(R|_{I_m \setminus B}) = \sigma^{-1}(F) = a = F(R).$$

Hence, F satisfies IIA.

Conversely, assume that F is Paretian and satisfies IIA. Again take any $m \in \mathbb{N}$, $R \in \mathscr{L}(I_m)^N$, and set $F(R) = a$. To show that F is self-selective at R, let $\mathscr{A}$ be any finite subset of $\mathscr{N}$ with $F \in \mathscr{A}$. Set $\mathrm{Im}_R \mathscr{A} = \{G(R) \mid G \in \mathscr{A}\}$. For each $x \in \mathrm{Im}_R \mathscr{A}$, let $\mathscr{A}_x = \{G \in \mathscr{A} \mid G(R) = x\}$. Now choose and fix one $H_x \in \mathscr{A}_x$ for each $x \in \mathrm{Im}_R \mathscr{A} \setminus \{a\}$, and let $H_a = F$. Note that there exists some $L \in \mathscr{L}(\mathscr{A}, R)$ such that $H_x L^i G$ for all $x \in \mathrm{Im}_R \mathscr{A}$, $G \in \mathscr{A}_x$ and $i \in N$. Denoting $\mathscr{B} = \{H_x \mid x \in \mathrm{Im}_R \mathscr{A}\}$, we see that $F(L) \in \mathscr{B}$ since F is Paretian. Since $F = H_a$, we also have $F \in \mathscr{B}$. Now by IIA, it follows that $F(L) = F(L|_{\mathscr{B}})$. Moreover, for each $y \in I_m \setminus \mathrm{Im}_R \mathscr{A}$, choose some $H_y \in \mathscr{N}$ such that $H_y(R) = \{y\}$, and set $\mathscr{A}' = \mathscr{B} \cup \{H_y \mid y \in I_m \setminus \mathrm{Im}_R \mathscr{A}\}$. Now it is clear that $\mathscr{L}(\mathscr{A}', R) = \{L'\}$, where $L'|_{\mathscr{B}} = L|_{\mathscr{B}}$. Defining the bijection $\sigma : I_m \to \mathscr{A}'$ by $\sigma(z) = H_z$ for each $z \in I_m$, we see that $R_{\sigma^{-1}} = L'$, which, in turn implies that

$$F(L') = F(R_{\sigma^{-1}}) = \sigma(F(R)) = \sigma(a) = H_a = F.$$

Finally, since F satisfies IIA and $L'|_{\mathscr{B}} = L|_{\mathscr{B}}$, we have

$$F(L') = F(L'|_{\mathscr{B}}) = F(L|_{\mathscr{B}}) = F(L).$$

Thus, $F(L) = F$. That is, F is self-selective at R relative to $\mathscr{A}$. However, since R and $\mathscr{A}$ were arbitrary, this means that F is universally self-selective. *Q.E.D.*

After having characterized universal self-selectivity for neutral unanimous SCFs through the above theorem, we will now show that such an SCF uniquely leads to a social welfare function on $\mathscr{L}(I_m)^N$ when we restrict the domain of our SCF to $\bigcup_{k \in I_m} \mathscr{L}(I_k)^N$ for any $m \in \mathbb{N}$. To do that we need to remind the reader of some definitions and well-known results.

Given a nomempty finite set A, a function $c : 2^A \setminus \{\varnothing\} \to 2^A \setminus \{\varnothing\}$ is called a *choice function* on A if and only if $c(X) \subset X$ for all $X \in 2^A \setminus \{\varnothing\}$. Let c be a choice function on A. c is said to satisfy *Sen's Property* α if and only if

$$\forall X, Y \in 2^A, \forall x \in X: \quad [X \subset Y \text{ and } x \in c(Y)] \Rightarrow x \in c(X).$$

We say that c satisfies *Sen's Property* β if and only if

$$\forall X, Y \in 2^A, \forall x, y \in c(X): \quad [X \subset Y \text{ and } y \in c(Y)] \Rightarrow x \in c(Y).$$

Finally, c is said to satisfy *Houthakker's Axiom* if and only if

$$\forall X, Y \in 2^A, \forall x, y \in X \cap Y: \quad [x \in c(X) \text{ and } y \in c(Y)] \Rightarrow x \in c(Y).$$

We know from Sen (1970) that the conjunction of Sen's Properties α and β is equivalent to Houthakker's Axiom. We also know from Houthakker (1950) that there exists a unique complete preorder $\succ$ on A such that $c(X) = \{x \in X \mid x \succ y$ for all $y \in X\}$ at each $X \in 2^A \setminus \{\emptyset\}$ if and only if c satisfies Houthakker's Axiom. In fact, if c is a choice function on A satisfying Houthakker's Axiom, then the complete preorder $\succ$ above is defined through the following biconditional: for any $x, y \in A$, one has $x \succ y$ if and only if $x \in c(\{x, y\})$.

Now let F be a neutral SCF and $m \in \mathbb{N}$. Given $R \in \mathscr{L}(I_m)^N$, the function $c_R : 2^{I_m} \setminus \{\emptyset\} \to 2^{I_m} \setminus \{\emptyset\}$ defined by $c_R(X) = \{F(R|_X)\}$ at each $X \in 2^{I_m} \setminus \{\emptyset\}$ is clearly a choice function on I_m. We will now show that c_R satisfies Houthakker's Axiom for each $R \in \mathscr{L}(I_m)^N$ if $F \in \mathscr{N}$ is universally self-selective.

PROPOSITION 1: *If $F \in \mathscr{N}$ is universally self-selective, then c_R satisfies Houthakker's Axiom for any $R \in \mathscr{L}(I_m)^N$, where $m \in \mathbb{N}$.*

PROOF: Assume that $F \in \mathscr{N}$ is universally self-selective. Let $m \in \mathbb{N}$ and take any $R \in \mathscr{L}(I_m)^N$. Also let $X, Y \in 2^{I_m}$ with $X \subset Y$, and assume that $x \in X \cap c_R(Y)$. By definition of c_R, $c_R(Y) = \{F(R|_Y)\}$. So, $F(R|_Y) = x$. From Theorem 1, we know that F satisfies IIA since F is universally self-selective. (Note that unanimity of F was not utilized to show that IIA is necessary for universal self-selectivity in Theorem 1.) Conjoined with the neutrality of F, this simply means that $F(R|_Y) = F(R|_{Y \setminus (Y \setminus X)}) = F(R|_X)$ since $F(R|_Y) = x \notin Y \setminus X$. But since $c_R(X) = \{F(R|_X)\}$, we conclude that $x \in c_R(X)$. So, c_R satisfies Sen's Property α. But Sen's Property β is also trivially satisfied by c_R since, for any $X \in 2^{I_m} \setminus \{\emptyset\}$ and any $x, y \in c_R(X)$, one must have $x = y$. Therefore, c_R satisfies Houthakker's Axiom. *Q.E.D.*

In view of Proposition 1, for any neutral universally self-selective SCF F and any $R \in \mathscr{L}(I_m)^N (m \in \mathbb{N})$, there exists a complete preorder $\succ_R$ on I_m such that, for all $X \in 2^{I_m} \setminus \{\emptyset\}$, $c_R(X) = \{x \in X \mid x \succ_R y$ for all $y \in X\}$. Since F is a social choice function and thus c_R is a singleton-valued choice function on I_m for each $R \in \mathscr{L}(I_m)^N$, $\succ_R$ is actually a linear order on I_m for each such R. Thus, associating the linear order $\succ_R$ with the linear order profile R yields a social welfare function via the SCF F. Formally, given any universally self-selective $F \in \mathscr{N}$ and $m \in \mathbb{N}$, we will call the function $f_F^m : \mathscr{L}(I_m)^N \to \mathscr{L}(I_m)$ defined by $f_F^m(R) = \succ_R$ at each $R \in \mathscr{L}(I_m)^N$ *the social welfare function induced by F on I_m*.

As usual, given a nonempty set A, we call a function from $\mathscr{L}(A)^N$ into $\mathscr{L}(A)$ a *social welfare function* (SWF). Thus, for any universally self-selective $F \in \mathscr{N}$

and $m \in \mathbb{N}$, the social welfare function induced by F on I_m is actually an SWF. Given an SWF $f: \mathscr{L}(A)^N \to \mathscr{L}(A)$, we say that f *satisfies Independence of Irrelevant Alternatives* (IIA) if and only if

$$\forall R, R' \in \mathscr{L}(A)^N, \forall B \in 2^A \setminus \{\varnothing\}: \quad R|_B = R'|_B \Rightarrow f(R)|_B = f(R')|_B;$$

and f is said to be *Paretian* if and only if

$$\forall R \in \mathscr{L}\big((I_m)^N, \forall a, b \in I_m: \quad [\forall i \in N: aR^i b] \Rightarrow af(R)b.$$

Remembering that a unanimous universally self-selective neutral SCF F is Paretian and satisfies IIA, we will now show that these latter properties are inherited by the SWF f_F^m induced by F on I_m for each $m \in \mathbb{N}$.

PROPOSITION 2: *Let $F \in \mathscr{N}$ be unanimous. If F is universally self-selective, then f_F^m is Paretian and satisfies IIA for each $m \in \mathbb{N}$.*

PROOF: Assume that F is universally self-selective. Now take any $m \in \mathbb{N}$. Let $R \in \mathscr{L}(I_m)^N$, $a, b \in I_m$, and assume that $aR^i b$ for all $i \in N$. Now since F is unanimous, $F(R|_{\{a,b\}}) = a$. But then $c_R(\{a,b\}) = \{a\}$, implying that $a \succ_R b$, i.e., $af_F^m(R)b$. Thus, f_F^m is Paretian.

Now let $\varnothing \neq B \subset I_m$, and take any $R, R' \in \mathscr{L}(I_m)^N$ with $R|_B = R'|_B$. Moreover, let $b, c \in B$. But then $R|_{\{b,c\}} = R'|_{\{b,c\}}$ and so $c_R(\{b,c\}) = F(R|_{\{b,c\}}) = F(R'|_{\{b,c\}}) = c_{R'}(\{b,c\})$. Therefore $bf_F^m(R)c$ if and only if $bf_F^m(R')c$, implying that $f_F^m(R)|_B = f_F^m(R')|_B$. *Q.E.D.*

Note that the hypothesis regarding the universal self-selectivity of F was only used to ensure that f_F^m is a well-defined SWF for all $m \in \mathbb{N}$. We now know by Arrow's Impossibility Theorem (Arrow (1963)) that f_F^m is dictatorial for each $m \in \mathbb{N}$ with $m \geq 3$. We will show below that the dictator who is the same agent for all $m \geq 3$ will also be the dictator for $m \in \{1,2\}$ utilizing the universal self-selectivity of F. Formally, an SCF $F: \bigcup_{m \in \mathbb{N}} \mathscr{L}(I_m)^N \to \mathbb{N}$ is said to be *dictatorial* if and only if

$$\exists i \in N, \forall m \in \mathbb{N}, \forall R \in \mathscr{L}(I_m)^N: \quad F(R) = \underset{I_m}{\operatorname{argmax}} R^i.$$

Given a finite nonempty set A and an SWF $f: \mathscr{L}(A)^N \to \mathscr{L}(A)$, we say that f is *dictatorial* if and only if

$$\exists i \in N, \forall R \in \mathscr{L}(A)^N: \quad f(R) = R^i.$$

THEOREM 2: *Let $F \in \mathscr{N}$ be unanimous. Now F is universally self-selective if and only if it is dictatorial.*

PROOF: The "if" part is obvious. Now assume that F is universally self-selective. Then we know that by Proposition 2 f_F^m is Paretian and satisfies IIA for all

$m \in \mathbb{N}$. But then f_F^m is dictatorial for each $m \geq 3$ by Arrow's Theorem. So, for each such m, there exists some $i_m \in N$ such that, for all $R \in \mathscr{L}(I_m)^N$, $f_F^m(R) = R^{i_m}$. Set $\operatorname{argmax}_{I_m} R^{i_m} = a$, and suppose that $F(R) = b$, where $b \neq a$. But now $a f_F^m(R) b$, i.e. $\{F(R|_{\{a,b\}})\} = c_R(\{a,b\}) = \{a\}$. On the other hand, F satisfies IIA since it is universally self-selective by Theorem 1. Thus, $F(R|_{\{a,b\}}) = b$, a contradiction. So, $F(R) = a = \operatorname{argmax}_{I_m} R^{i_m}$ for all $R \in \mathscr{L}(I_m)^N$.

Now consider any $k > l \geq 3$. Let $R \in \mathscr{L}(I_k)^N$ be defined as follows: For any $t \in I_{k-1}$; $tR^{i_k}(t+1)$, and $(t+1)R^j t$ for all $j \in N \setminus \{i_k\}$. Then $F(R) = \operatorname{argmax}_{I_k} R^{i_k} = 1$. On the other hand, $F(R|_{I_l}) = 1$ again since F satisfies IIA. But, for each $j \in N \setminus \{i_k\}$, $\operatorname{argmax}_{I_l} R|_{I_l} = l \neq 1$. Thus, $i_k = i_l$. In summary, there is some $i_0 \in N$ such that

$$\forall m \in \mathbb{N}, \forall R \in \mathscr{L}(I_m)^N: \quad m \geq 3 \Rightarrow F(R) = \operatorname*{argmax}_{I_m} R^{i_0}.$$

Finally, take any $R \in \mathscr{L}(I_2)^N$. Define $R' \in \mathscr{L}(I_3)^N$ as follows: for any $i \in N$ and any $x, y \in I_2$, $xR'^i y$ if and only if $xR^i y$; and for any $i \in N$ and $x \in I_2$, $xR'^i 3$. Then $F(R') \in I_2$ since F is Paretian by Theorem 1 and $F(R') = \operatorname{argmax}_{I_3} R^{i_0}$. But since F also satisfies IIA and $R'|_{I_2} = R$, we have $F(R') = F(R)$. Moreover, by construction of R', $\operatorname{argmax}_{I_3} R'^{i_0} = \operatorname{argmax}_{I_2} R^{i_0}$, implying that $F(R) = \operatorname{argmax}_{I_2} R^{i_0}$. Since i_0 is trivially a dictator when $m = 1$, we conclude that F is dictatorial. *Q.E.D.*

Now we can easily obtain from Theorem 2 new characterizations of universal self-selectivity for neutral unanimous SCFs in terms of strategy-proofness and monotonicity. But we first need to extend the latter two notions to SCFs in our context, and we will do so by proceeding "componentwise." Given an SCF $F \in \mathscr{F}$, for each $m \in \mathbb{N}$, we let $F_m : \mathscr{L}(I_m)^N \to I_m$ be the restriction of F to $\mathscr{L}(I_m)^N$. Moreover, as usual, we say that F_m is *monotonic* if and only if

$$\forall R, R' \in \mathscr{L}(I_m)^N:$$
$$\left[(\forall i \in N, \forall x \in I_m : F_m(R) R^i x \Rightarrow F_m(R) R'^i x) \Rightarrow F_m(R) = F_m(R')\right]$$

and F_m is said to be *strategy-proof* if and only if

$$\forall R \in \mathscr{L}(I_m)^N, \forall i \in N, \forall R'^i \in \mathscr{L}(I_m): \quad F_m(R) R^i F_m(R^{N \setminus \{i\}}, R'^i).$$

Finally, we say that an SCF $F \in \mathscr{F}$ is *monotonic* if and only if F_m is monotonic for all $m \in \mathbb{N}$ and, similarly, F will be called *strategy-proof* if and only if F_m is strategy-proof for each $m \in \mathbb{N}$.

If a unanimous $F \in \mathscr{N}$ is universally self-selective, then it is dictatorial by Theorem 2, from which it trivially follows that F is both monotonic and strategy-proof. A unanimous SCF $F \in \mathscr{N}$ that is monotonic or strategy-proof need not be universally self-selective, however. Let F be a neutral unanimous and monotonic SCF, for example. It is true that then, for each $m \geq 3$, F_m will be dictatorial by the Müller-Satterthwaite (1977) Theorem. Not only because F_2 is not necessarily dictatorial, but also since the dictators for different values of $m \geq 3$ need not coincide, F may not be dictatorial and thus not universally self-selective. To give a specific example of this, assume that $a, b \in N$ with $a \neq b$ and define F to be the SCF that is dictatorial on $\mathscr{L}(I_m)^N$ for each $m \in \mathbb{N}$, where the dictator is a if m is odd while the dictator is b for all even m. F is clearly neutral unanimous and monotonic, but it is also easily seen not to be universally self-selective. The same example shows that we have a similar situation when monotonicity is replaced by strategy-proofness above.

The obvious reason for the above phenomenon is that monotonicity as well as strategy-proofness of an SCF F treats the components F_m of F separately independent of each other. The condition that provides the desired interdependence between the components of F in the sense that it implies universal self-selectivity when conjoined with either monotonicity or strategy-proofness is IIA. We state and prove this result in the following corollary.

COROLLARY 1: *Let $F \in \mathscr{N}$ be unanimous.*
1. *F is universally self-selective if and only if F is monotonic and satisfies IIA.*
2. *F is universally self-selective if and only if F is strategy-proof and satisfies IIA.*

PROOF: As the proofs of (1) and (2) are similar, we will only prove (1). The "only if" part of (1) follows from Theorems 1 and 2. Now assume that F is monotonic and satisfies IIA. Then F_m is monotonic for each $m \in \mathbb{N}$. But then, for all $m \geq 3$, F_m is dictatorial by the Müller-Satterthwaite (1977) Theorem, since F_m clearly also satisfies citizen sovereignty because it is neutral (i.e., for each $k \in I_m$, there exists some $R \in \mathscr{L}(I_m)^N$ with $F_m(R) = k$). Now, as in the proof of Theorem 2, IIA implies that the dictator, which must be the same for all $m \geq 3$, is also a dictator for $m = 2$. In the proof of (2), the Gibbard (1973)–Satterthwaite (1973) Theorem is used instead of the Müller-Satterthwaite (1977) Theorem. *Q.E.D.*

4. CONCLUSIONS

Here we have found another set of properties for SCFs (which in our context are entire classes of social choice functions in the standard sense indexed by natural numbers) resulting in dictatoriality; namely a neutral unanimous SCF turns out to be dictatorial if it is universally self-selective as well. A naturally arising concern to be addressed now is that neutrality conjoined with unanimity and single-valuedness might already be narrowing down the class of social choice rules to such an extent that not much is left to the concept of self-selectivity to further reduce it to just dictatorial ones. The best way of dealing with this concern is, of course, to simply compute the cardinality of the class of unanimous neutral SCFs.

If our society N consists of n agents, then the number of dictatorial SCFs is just n although there clearly are infinitely many neutral unanimous SCFs. The infinite cardinality of this class may, however, still not be reflecting the existence of a broad spectrum of such SCFs. Every assignment of an agent $i_m \in N$ as a dictator on $\mathscr{L}(I_m)^N$ for each $m \in \mathbb{N}$, for example, yields a unanimous neutral SCF, and there are obviously infinitely many such SCFs. However, it is impossible to claim that this class represents a rich variety of structure on the part of SCFs of the desired kind. What self-selectivity additionally imposes upon members of this class simply consists of requiring that one has to be consistent with the choice of the dictators irrespective of the size of the alternative set, shrinking the set of admissible sequences of dictators to constant ones. Thus, a more detailed examination of this problem is needed.

Referring to a function $F: \mathscr{L}(I_m)^N \to I_m$ as an SCF of order m for each $m \in \mathbb{N}$, we now will compute the number of unanimous neutral SCFs of order m. Now take any $m \in \mathbb{N}$ and set $|N| = n$. To first find the number of unanimous neutral SCFs of order m, note that the assignment of a member of I_m to a linear order profile R in $\mathscr{L}(I_m)^N$ uniquely determines what alternatives from I_m should be assigned to linear order profiles that can be obtained from R by a permutation on I_m as well, for the resulting SCF to be neutral. On the other hand, neutrality imposes no restrictions upon the choice of alternatives to be assigned to two linear order profiles that cannot be obtained from each other via such a permutation.

To formalize this observation, for any $R, R' \in \mathscr{L}(I_m)^N$, we define $R \sim R'$ if and only if $R' = R_\sigma$ for some permutation σ on I_m. Now $\sim$ is clearly an equivalence relation on $\mathscr{L}(I_m)^N$. Moreover, each equivalence class of $\sim$ contains exactly $m!$ elements since there are $m!$ permutations on I_m. Denoting the quotient set of $\mathscr{L}(I_m)^N$ with respect to $\sim$ by $\mathscr{L}(I_m)^N/\sim$ as usual (which is defined as the set of all equivalence classes of $\sim$), it is a direct consequence of our observation above that the number of neutral SCFs of order m is nothing but the number of all functions $f: \mathscr{L}(I_m)^N/\sim \to I_m$. But clearly $|\mathscr{L}(I_m)^N| = (m!)^n$, so that $|\mathscr{L}(I_m)^N/\sim| = (m!)^n/m! = (m!)^{n-1}$, and thus the desired number of functions is $m^{(m!)^{n-1}}$.

Now we can turn to the problem of finding the number of unanimous neutral SCFs of order m. We call a linear order profile $R \in \mathscr{L}(I_m)^N$ *unanimous* if and only if there is some $k \in I_m$ such that k is the top alternative of R^i for each $i \in N$. Notice that, in any equivalence class of $\sim$, either all linear order profiles are unanimous or none is so. Now the number of unanimous profiles in $\mathscr{L}(I_m)^N$ is clearly equal to $m((m-1)!)^n$, and thus the number of equivalence classes in $\mathscr{L}(I_m)^N/\sim$ consisting of unanimous profiles only is $m((m-1)!)^n/m! = ((m-1)!)^{n-1}$. Now since the outcome a unanimous SCF assigns to a unanimous linear order profile is uniquely determined as the alternative unanimously top ranked at that profile, the number of unanimous neutral SCFs of order m is nothing but the number of all functions that take their values in I_m and whose domain consists of exactly those equivalence classes in $\mathscr{L}(I_m)^N/\sim$ that contain no unanimous profiles. The number of such functions is

$$m^{(m!)^{n-1} - ((m-1)!)^{n-1}} = m^{(m^{n-1}-1)((m-1)!)^{n-1}}.$$

When $n = 3$ and $m = 2$, the number of unanimous neutral SCFs is eight, and from among these, three are dictatorial. As m increases (with n kept fixed), this number increases very rapidly, however. Already when $n = 3$ and $m = 4$, it becomes

$$4^{(4^2-1)(3!)^2} = 4^{15 \times 36} = 16^{225},$$

exceeding 10^{80} by far, which is the estimated order of magnitude of the total number of elementary particles in our universe, while again only three from among these are dictatorial. This estimation seems to shed sufficient light on the problem so as to allow us to judge the role the self-selectivity plays in narrowing down the class of unanimous neutral SCFs to dictatoriality.

The proof of our main result here is based upon the observation that a unanimous neutral and universally self-selective SCF leads to a class of neutral Paretian social welfare functions satisfying IIA, each defined on the set of linear order profiles on an initial segment of natural numbers. Now it can also easily be seen that the SCF with which one starts can be obtained back from such a class of social welfare functions in a unique fashion. It is this "isomorphism" between these two objects that allows us to conclude the restriction of Binmore's (1975) result to Paretian social welfare functions defined on linear order profiles on a three element alternative set as a corollary here and to extend it to the case where the number of alternatives is any positive integer k.

A natural question now is whether we can escape the pessimistic conclusion of the paper by relaxing some of our hypotheses, i.e., whether we can have universally self-selective nondictatorial social choice rules that may not satisfy some of the other conditions we assumed here. Neutrality seems to be both natural and essential for the kind of consistency we deal with in this paper. As unanimity of our SCFs corresponds to the Paretianism of social welfare functions they induce, in the light of Wilson's (1972) version of Arrow's Theorem without the Pareto Principle, the conjecture is that the deletion of the hypothesis about unanimity will broaden the class of neutral universally self-selective SCFs by only including anti-dictatorships along with dictatorships into this class, as is also suggested by Binmore's (1975) example, so long as we confine our agents' preferences to linear orders. The two main ways that remain to possibly escape dictatoriality still preserving consistency in the sense of self-selectivity seem to be either restricting the domains of the SCFs with which we deal or allowing our social choice rules to be multi-valued.

Koray (1998) considers a combination of these two possibilities in the context of electoral system design. A voting rule there is defined to be a nonempty-valued neutral and top-majoritarian social choice correspondence (SCC), where an SCC is said to be *top-majoritarian* if and only if, at all profiles where there is a strict majority top-ranking an alternative, it chooses the singleton consisting of that alternative only. A dictatorial SCC is clearly not a voting rule according to this definition for it is not top-majoritarian. However, the notion of self-selectivity employed in Koray (1998) for SCCs again is relative to finite sets of neutral SCCs that contain the voting rule considered, but whose other members need not be top-majoritarian. In other words, for a voting rule to be self-selective at a given preference profile on an alternative set, it is required to choose itself at each induced profile also in the presence of dictatorial social choice rules. As one might easily guess in the light of our results here, it turns out that there are no voting rules that are universally self-selective. It, however, also turns out that it is exactly the linear order profiles with no Condorcet winners at which self-selective voting rules fail to exist. Thus, confining ourselves to linear order profiles at which Condorcet winners do exist also guarantees the existence of self-selective voting rules. In fact, the Condorcet rule itself turns out to be self-selective at all such profiles. Moreover, as any voting rule that is self-selective at such preference profiles is shown to be a refinement of the Condorcet rule, Koray (1998) rediscovers the Condorcet rule as the maximal neutral and self-selective social choice rule. Not every nonempty-valued refinement of the Condorcet rule is self-selective at all linear order profiles with Condorcet winners, however, for such a refinement need not satisfy IIA.

The characterization of self-selectivity for the broader class of nonempty-valued neutral and unanimous (rather than top-majoritarian) SCCs also seems to be an interesting problem that is yet to be done.

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